Mathematics for Machine Learning: Homework 4
Deadline is 13.08.2020

August 5, 2020

1. Find the geometric and algebraic multiplicity of the eigenvalue(s) of the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

2. Let $A \in \mathbb{R}^{3\times 3}$ be a matrix such that
$$A\mathbf{v}_1 = 3\mathbf{v}_1,\ A\mathbf{v}_2 = 5\mathbf{v}_2,\ A\mathbf{v}_3 = -\mathbf{v}_3,$$
for some non-zero vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$. Determine the eigenvalues, characteristic polynomial, determinant and the trace of the matrix A. Find the characteristic polynomial of A^T.

3. Compute the trace of the matrix $ABCB^{-1}A^{-1}$ given that $A, B, C \in \mathbb{R}^{5\times 5}$ with $C\mathbf{v}_i = 2^i\mathbf{v}_i, i = 1, \ldots, 5$ for some non-zero vectors $\mathbf{v}_1, \ldots, \mathbf{v}_5 \in \mathbb{R}^5$.

4. Find the Cholesky decomposition of the 3×3 matrix
$$A = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 8 & 12 \\ -3 & 12 & 27 \end{bmatrix}.$$

5. Compute the eigendecomposition of a (symmetric) matrix
$$A = \begin{bmatrix} 0 & 4 \\ 4 & 6 \end{bmatrix}.$$

6. Find the SVD of the matrix

   a) $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$,

   b) $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$.

7. Using the definition of limit, prove that

   a) $\displaystyle\lim_{n\to\infty} \frac{2 + (-1)^n}{n} = 0$,

   b) $\displaystyle\lim_{n\to\infty} \frac{3n\sin n + 1}{2n^2 + 2n - 1} = 0$,

   c) $\displaystyle\lim_{n\to\infty} \frac{2n^2 + 1}{8n^2 - 2n + 10} = \frac{1}{4}$,

   d) $\displaystyle\lim_{n\to\infty} \frac{\sqrt{3n^2 + 1}}{\sqrt{n^2 + 2n + 10}} = \sqrt{3}$.

8. Prove that the sequence x_n is divergent

a) $x_n = \dfrac{n}{n+1} \cos \dfrac{2\pi n}{3}$,

b) $x_n = 2^{(-1)^n n}$,

c) $x_n = \dfrac{n^2 - 2n}{n+1}$,

d) $x_n = n^2 \sin \dfrac{\pi n}{4}$.