Mathematics for Machine Learning: Homework 6
Deadline is 27.08.2020

August 20, 2020

1. Calculate the integral

   a) $\int_0^{2\pi} \cos^2 2x\,dx,$

   b) $\int_0^1 \frac{x^2}{1+x^6}\,dx,$

   c) $\int_0^{\ln 2} \frac{dx}{\sqrt{1+e^x}},$

   d) $\int_{\frac{1}{e}}^e |\ln x|\,dx.$

2. Calculate the improper integral

   a) $\int_0^1 \ln x\,dx,$

   b) $\int_0^{+\infty} x2^{-x}\,dx,$

   c) $\int_2^{+\infty} \frac{dx}{x^2+x-2},$

   d) $\int_0^{+\infty} \frac{x\ln x}{(1+x^2)^2}\,dx.$

3. Study the convergence of the improper integral

   a) $\int_1^{+\infty} \frac{dx}{x\sqrt[3]{x^2+1}},$

   b) $\int_1^{+\infty} \frac{\operatorname{tg}\frac{1}{x}}{1+x\sqrt{x}}\,dx,$

   c) $\int_0^2 \frac{\ln\left(1+\sqrt[5]{x^3}\right)}{e^{\sin x}-1}\,dx,$

   d) $\int_0^1 \frac{e^x}{\sqrt{1-x^3}}\,dx.$

4. Find the Taylor series representation of the function f at the point $x_0 = 0$.

   a) $f(x) = \cos^2 x,$

   b) $f(x) = \frac{x}{1+x-2x^2}.$

5. Calculate the gradient of f.

    a) $f(x, y) = x \sin(x + y)$,

    b) $f(x, y) = \text{argtg} \dfrac{x + y}{1 - xy}$,

    c) $f(x, y, z) = xyze^{x+y+z}$,

    d) $f(x, y, z) = x^{\frac{y}{z}}$.

6. Find $\dfrac{\partial u}{\partial t}$ and $\dfrac{\partial u}{\partial s}$, if

    a) $u = f(x)$, $x = t^2 + s^2$,

    b) $u = f(x, y)$, $x = \sin t$, $y = \cos s$,

    c) $u = f(x)g(y)$, $x = ts$, $y = t - s$,

    d) $u = f(x, y, z)$, $x = ts$, $y = 2t + s$, $z = t - 3s$.