Mathematics for Machine Learning

Lab 5

Problem 1. Show the equality using the definition of limit for functions.

1) $\lim_{x\to 1} \frac{3x^2-4x+1}{x-1} = 2$ **2)** $\lim_{x\to 3} \frac{x^3-3x^2}{x-3} = 9$

Problem 2. Calculate the limit of $f(x)$ function, when $x \to a$.

1) $f(x) = \frac{sinx}{x}, \quad a = \infty$

2) $f(x) = \frac{tgx}{x}, \quad a = 0$

3) $f(x) = \frac{1-cosx}{x^2}, \quad a = 0$

4) $f(x) = \frac{\sqrt{1-cosx^2}}{1-cosx}, \quad a = 0$

5) $f(x) = \left(\frac{x+2}{x-2}\right)^x, \quad a = \infty$

6) $f(x) = \frac{\ln(x^2+e^x)}{1+xe^x}, \quad a = 0$

Problem 3. Check whether the given functions are continuous on $\mathbb{R}$.

1) $f(x) = \begin{cases} \frac{x^3-64}{x^2-16}, & x \neq 4 \\ 6, & x = 4 \end{cases}$

2) $f(x) = \begin{cases} \frac{x^2-10}{5-x}, & x < 0 \\ 2, & x = 0 \\ \sqrt{4+x^2}, & x > 0 \end{cases}$

Problem 4. Find the extremum points and their corresponding values for the given functions.

1) $f(x) = x^3 - 6x^2 + 9x - 4$

2) $f(x) = \frac{2x}{1+x^2}$

3) $f(x) = \frac{ln^2x}{x}$

4) $f(x) = \frac{10}{1+sin^2x}$

Problem 5. Find the extremum values of $f(x)$ on the specified interval.

1) $f(x) = x^4 + 32x + 1,\ x \in [-2; 0]$

2) $f(x) = x^3 - 6x^2 + 9x - 1,\ x \in [-1; 4]$

3) $f(x) = \sqrt{5-4x},\ x \in [-1; 1]$

4) $f(x) = \sqrt{x} - \sqrt{x^3}$, $x \in [0, 4]$

Problem 6. Find the set of antiderivative (primitive) functions F, such that $F'(x) = f(x)$, where

1) $f(x) = \sqrt[3]{2x}$

2) $f(x) = x^2(x^2 - 3)$

3) $f(x) = ctg^2 x$

4) $f(x) = \dfrac{3 \cdot 2^x - 2 \cdot 3^x}{2^x}$

5) $f(x) = \ln x$

6) $f(x) = e^x \cdot sin(e^x)$

Problem 7. Check whether $\sum\limits_{n=1}^{\infty} a_n$ is convergent, if.

1) $a_n = \dfrac{n+2}{\sqrt[3]{n^3 + 2n + 4}}$

2) $a_n = \dfrac{1}{2^n + n}$

3) $a_n = \dfrac{1}{\sqrt{(2n-1)(2n+1)}}$

4) $a_n = \dfrac{(2n+1)!!}{3^n n!}$