

Mathematics for Machine Learning

Lab 10

Problem 1. Compute $\iint_D f(x, y) dx dy$ if

- 1) $f(x, y) = x^2 y, \quad D = [0, 3] \times [1, 2]$
- 2) $f(x, y) = \sin(2x + y), \quad D = [0, \pi] \times [0, 1]$
- 3) $f(x, y) = x + 2y, \quad D = \{(x, y) \mid -1 \leq x \leq 1, \quad 2x^2 \leq y \leq 1 + x^2\}$
- 4) $f(x, y) = x^2 + y^2, \quad D = \{(x, y) \mid 0 \leq y \leq 4, \quad \frac{y}{2} \leq x \leq \sqrt{y}\}$

Problem 2. Suppose that $X_n, (n \in \mathbb{N})$ is a sequence of uniformly distributed random variables on $(\frac{1}{n}, 2)$ and X r.v. is uniformly distributed in $(0, 2)$. Show that X_n converges to X in distribution.

Problem 3. Suppose X is a r.v. and $X_n = X + Y_n$, where $\mathbb{E}Y_n = \frac{1}{n}$ and $\text{Var}(Y_n) = \frac{\sigma^2}{n}$ ($\sigma > 0$). Show that X_n converges to X in probability.

Problem 4. Suppose the joint PDF of X and Y is the following

$$f_{X,Y}(x, y) = \begin{cases} ax + 1, & \text{if } x, y \geq 0, x + y < 1 \\ 0, & \text{if otherwise} \end{cases}$$

Find

- 1) the constant a
- 2) the marginal PDFs $f_X(x)$ and $f_Y(y)$
- 3) $P(Y < 2X^2)$

Problem 5. Suppose the joint PDF of X and Y is the following

$$f_{X,Y}(x, y) = ce^{-\frac{(x^2+y^2)}{2}}, \quad (x, y) \in \mathbb{R}^2$$

- 1) Find the constant c .
- 2) Find the marginal PDFs of X and Y
- 3) Study the dependence of X and Y .

Problem 6. Suppose that in Victory Park (Monument) the average time people wait in line to buy a ticket for the "Devil's Wheel" attraction is 10 minutes and the average time they wait to take the ride is 5 minutes. Assuming that the waiting times are independent, find the probability that a person waits a total of less than 20 minutes before taking the ride.